MULTI-AXLE LOAD IDENTIFICATION IN LABORATORY

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Purposes

- Present two methods on moving axle load identification
- Evaluate effects of various parameters on two methods
- Assess feasibility and robustness of two solutions, which are involved in two methods

Identification Methods

1) Equation of Motion

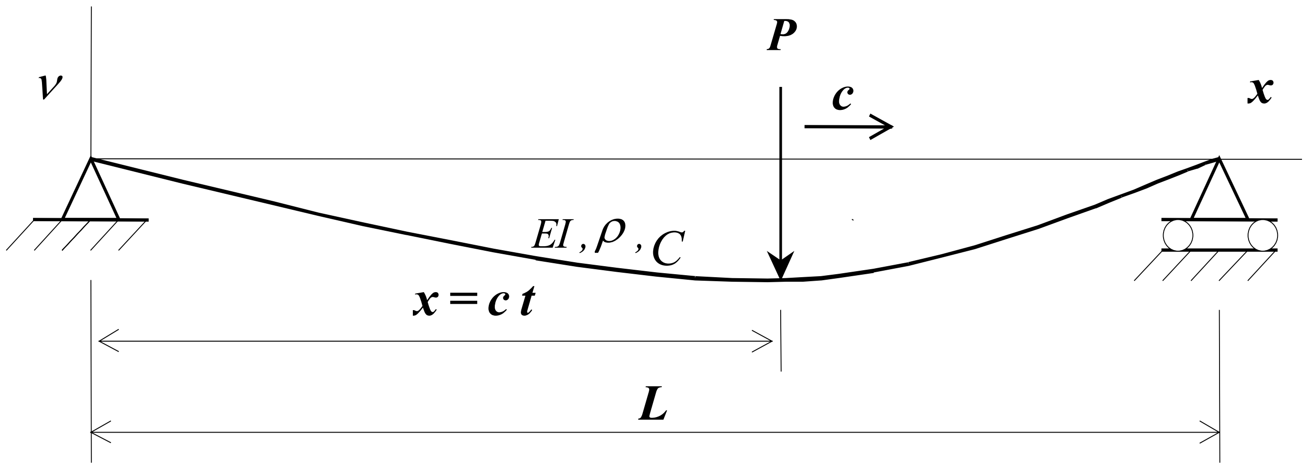


Figure 1. Moving forces on beam bridges

$$\rho \frac{\partial^2 v(x,t)}{\partial^2 t} + C \frac{\partial v(x,t)}{\partial t} + EI \frac{\partial^4 v(x,t)}{\partial x^4} = \delta(x-ct)P(t) \qquad (1)$$

$$\ddot{q}_n(t) + 2\xi_n \omega_n \dot{q}(t) + \omega_n^2 q_n(t) = \frac{2}{\rho L} p_n(t) \qquad (n = 1,2,\cdots\infty) \qquad (2)$$

Identification Methods

2a) Time Domain Method (**TDM**)

Modal Displacement:

$$q_n(t) = \frac{2}{\rho L}\int_0^t h_n(t-\tau)p(\tau)d\tau \qquad (3)$$

Dynamic Deflection:

$$v(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L} q_n(t) \qquad (4)$$

$$v(x,t) = \sum_{n=1}^{\infty} \frac{2}{\rho L \omega_n'} \sin\frac{n\pi x}{L}\int_0^t e^{-\xi_n \omega_n (t-\tau)} \sin\omega_n'(t-\tau)\sin\frac{n\pi c\tau}{L} P(\tau)\,d\tau \qquad (5)$$

Identification Methods

2b) Time Domain Method (**TDM**)

Bending moment *m (x,t)* in time domain *is*

$$m(x,t) = -EI\frac{\partial^2 v(x,t)}{\partial x^2}$$

$$= \sum_{n=1}^{\infty}\frac{2EI\pi^2 n^2}{\rho L^3 \omega_n'}\sin\frac{n\pi x}{L}\int_0^t e^{-\xi_n\omega_n(t-\tau)}\sin\omega_n'(t-\tau)\sin\frac{n\pi c\tau}{L}P(\tau)\,d\tau \tag{6}$$

$$B_{N\times N_B}\ P_{N_B\times 1} = R_{N\times 1} \tag{7}$$

Then, moving axle load *P(t)* can be identified by solving simultaneous equations (7) in time domain.

Identification Methods

3a) Frequency-Time Domain Method (**FTDM**)

Dynamic deflection in Eq.(4) in frequency domain is

$$V(x,\omega)=\sum_{n=1}^{\infty}\frac{2}{\rho L}\Phi_n(x)\,H_n(\omega)\,P(\omega) \qquad (8)$$

Here,

$$H_n(\omega)=\frac{1}{\omega_n^2-\omega^2+2\xi_n\omega_n\omega}$$

$$\Phi_n(x)=\sin(n\pi x/L)$$

$$P(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}p_n(t)\,e^{-i\omega t}\,dt$$

Identification Methods

3b) Frequency-Time Domain Method (FTDM)

Eq. (8) can be rearranged as

$$A_{(N+2)\times(N+2)}F_{(N+2)\times 1} = V_{(N+2)\times 1} \qquad (9)$$

Similarly, bending moment (R) in frequency domain is

$$D_{N\times N_B}P_{N_B\times 1} = R_{N\times 1} \qquad \mathbf{(10)}$$

$$P(\omega) \rightarrow P(t) \qquad \mathbf{(11)}$$

Identification Methods

4a) Solutions

Equations (7) and (10) become:

$$Ax = b \qquad (12)$$

Where,

A--- system matrix, known
b--- response vector, known
x--- force vector, unknown

Identification Methods

4b) Solutions

- **Pseudo Inverse (PI) solution**

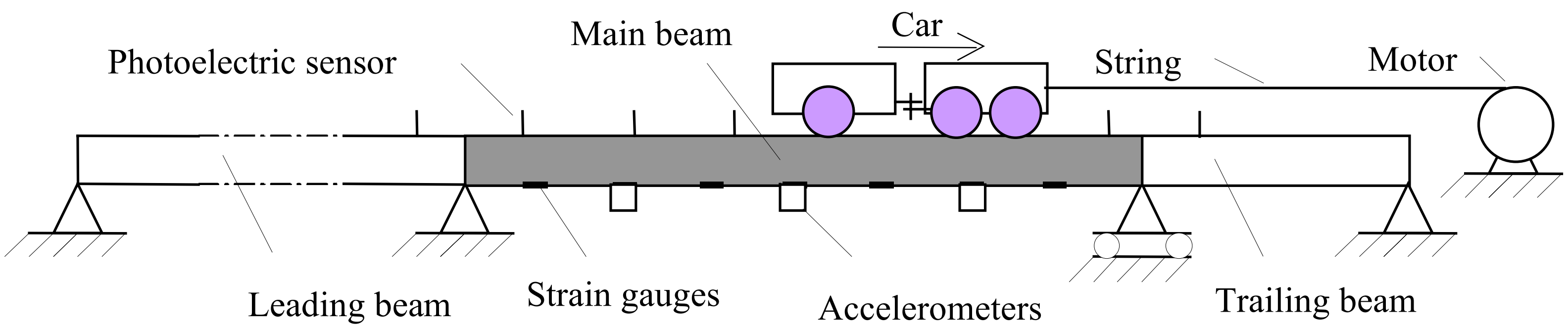
$$x = A^{+} b = [(A^{T} A)^{-1} A^{T}]\, b \qquad (13)$$

- **SVD solution** (*if* $A = USV^{T}$)

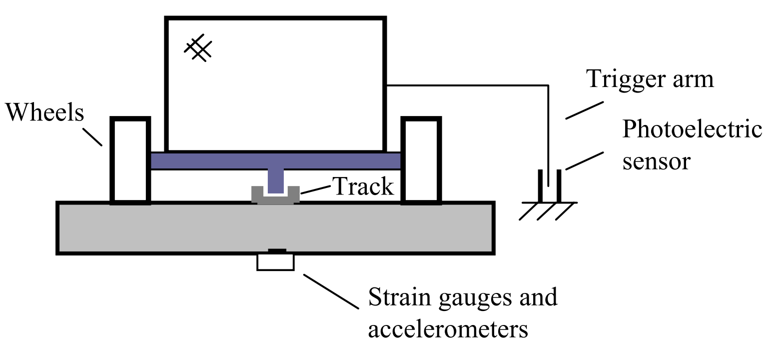
$$x = (VS^{-1}U^{T})\, b \qquad (14)$$

Experiments in Laboratory

1) Setup



View A-A



Vehicle:
ASSR=0.15:0.15/0.15:0.20
Axle spacing=0.56:0.56/0.56:0.75 m
Total weight=18 (2:8:8)/17(3:7:7) kg

Bridge (simply supported):
Size: 3678 x 101 x 25 mm
Material: mild steel

Multi-axle Load Identification

1) Definition of Error

Relative Percentage Error (*RPE*):

$$RPE = \frac{\sum \left| f_{true} - f_{identified} \right|}{\sum \left| f_{true} \right|} \times 100\% \qquad \textbf{(15)}$$

$$f_{true} \Leftrightarrow R_{measured}$$
$$f_{identified} \Leftrightarrow R_{rebuilt}$$

Accepted Tolerance:

RPE<10%

Multi-axle Load Identification

2) Study Scheme

- **Aim at evaluating effects of parameters on TDM and FTDM**
- **Parameters:**
  - Mode number of bridge
  - Measurement stations
  - Vehicle frame
  - Suspension system

Multi-axle Load Identification

3.1) Effect of Mode Number (MN)

Method	MN	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
TDM	5	10.86 <u>10.87</u>	10.32 <u>10.34</u>	25.66 <u>25.64</u>	8.55 <u>8.56</u>	2.80 <u>2.80</u>	3.58 <u>3.57</u>	11.97 <u>11.99</u>
	6	16.14 <u>16.14</u>	6.88 <u>6.88</u>	27.83 <u>27.82</u>	8.19 <u>8.20</u>	3.47 <u>3.46</u>	7.69 <u>7.69</u>	15.58 <u>15.57</u>
	7	19.31 <u>19.31</u>	8.50 <u>8.50</u>	25.14 <u>25.13</u>	6.16 <u>6.16</u>	6.01 <u>6.01</u>	7.08 <u>7.08</u>	15.53 <u>15.53</u>
FTDM	5	8.44 <u>115.00</u>	8.97 <u>81.28</u>	24.29 <u>74.61</u>	7.71 <u>42.35</u>	3.15 <u>52.17</u>	4.07 <u>78.31</u>	3.89 <u>119.40</u>
	6	7.28 <u>7.50</u>	7.78 <u>7.79</u>	24.23 <u>24.18</u>	7.72 <u>7.74</u>	4.92 <u>4.95</u>	4.22 <u>4.27</u>	5.46 <u>5.59</u>
	7	7.06 <u>7.36</u>	7.53 <u>7.61</u>	24.02 <u>23.95</u>	7.13 <u>7.13</u>	4.26 <u>4.29</u>	3.33 <u>3.40</u>	3.52 <u>3.67</u>

Note:Underlined values for PI, others for SVD.

<u>Remarks</u>

- **For TDM, no difference either using PI or SVD. Accuracy increases with MN. The worst occurs at 3rd station.**
- **For FTDM, SVD clearly better than PI. Accuracy independent of MN after MN=5. The worse occurs at 3rd station. FTDM better than TDM.**

Multi-axle Load Identification

3.2a) Effect of Measurement Stations

Method	MN	RPE (%)					
		Sta.1	Sta.2	Sta.4	Sta.5	Sta.6	Sta.7
TDM	5	5.19	3.92	1.97	2.46	2.50	5.20
	6	7.81	3.04	2.26	2.78	3.46	8.81
	7	9.08	3.81	3.01	2.33	3.47	8.74
FTDM	5	2.44	1.58	1.19	1.60	2.27	3.15
	6	2.04	1.58	1.26	1.56	1.76	2.43
	7	1.95	1.50	1.16	1.39	1.68	2.36

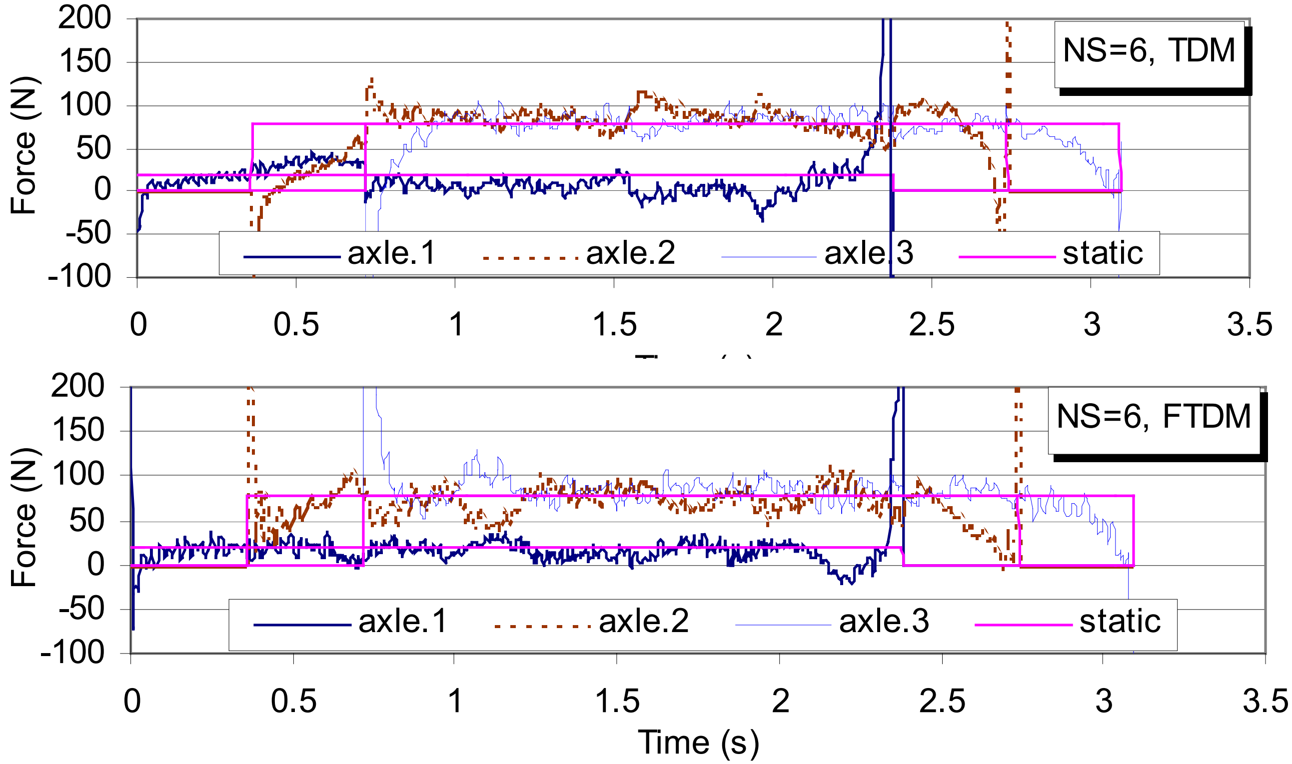
Note: Only for SVD.

Remarks

- **After elimination of 3rd station, accuracy are very much improved.**
- **All RPE values are less than 10%, FTDM is better than TDM.**

Multi-axle Load Identification

3.2b) Effect of Measurement Stations

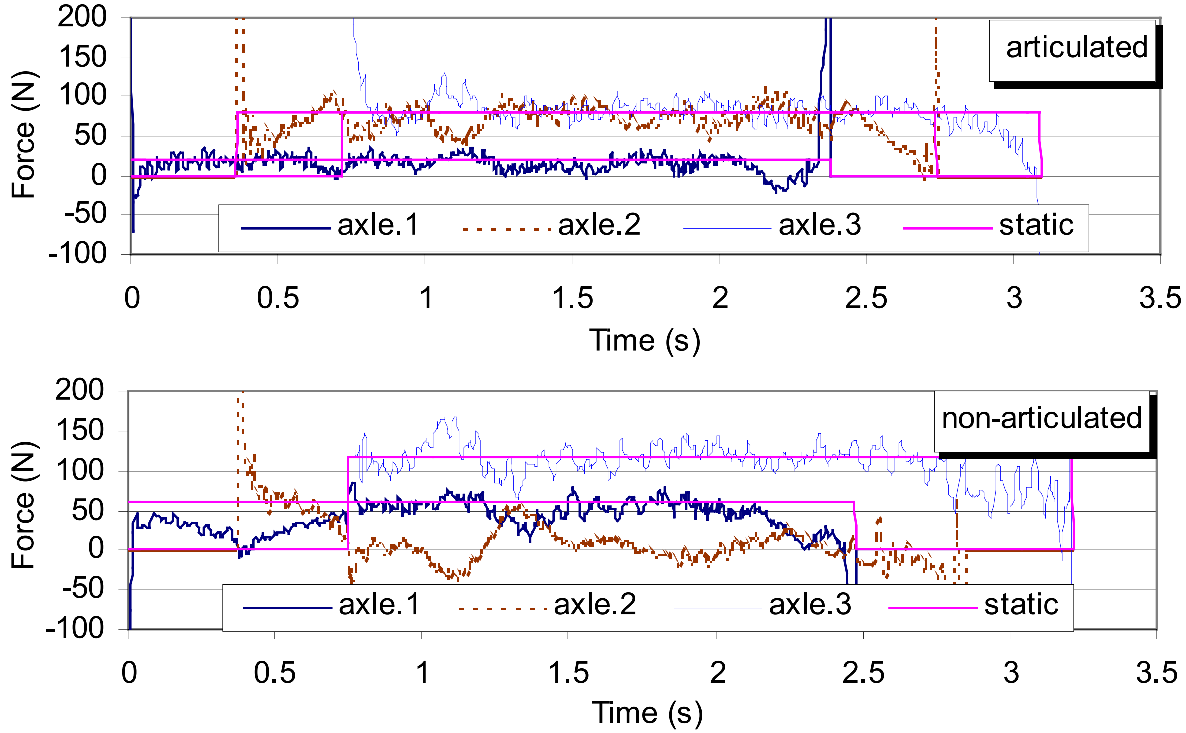


Remarks

- **The identified three-axle loads are better and reasonable, FTDM better than TDM. Elimination of significant error data is appropriate.**

Multi-axle Load Identification

3.3) Effect of Vehicle Frame



Remarks

- **The identified three-axle loads are better and reasonable.**
- **The identified results are correct even the second axle is hanging in the air.**

Multi-axle Load Identification

3.4) Effect of Suspension System

Vehicle	Method	RPE (%)					
		Sta.1	Sta.2	Sta.4	Sta.5	Sta.6	Sta.7
288NA, 31.27 *Hz* Rigid connection	TDM	5.86	6.38	4.45	4.31	5.06	9.82
	FTDM	14.02	8.04	6.90	6.48	7.80	16.85
288NAS3, 30.2 *Hz* Suspend at 3rd axle	TDM	4.85	5.34	3.55	3.08	3.96	6.82
	FTDM	4.33	5.00	3.15	3.07	4.25	7.06
288NAS23, 14.15 *Hz* Suspend at 2nd and 3rd axles	TDM	4.45	4.47	3.25	3.09	3.13	4.99
	FTDM	3.90	3.88	2.69	3.03	3.53	4.65
288NAP3, 10.97 *Hz* Suspend at 2nd and 3rd axles	TDM	7.86	5.62	3.20	2.75	2.85	5.23
	FTDM	4.11	5.04	2.75	2.61	2.90	4.56

Note: Underlined for pre-compressed spring case..

Remarks

- **Fundamental frequency decrease with increment of suspension.**
- **Accuracy increase with the suspension. FTDM better than TDM.**
- **Both TDM & FTDM can be efficiently applied to multi-axle load identification.**

Conclusions

- Both TDM and FTDM methods have been successfully applied to the multi-axle load identification. They can efficiently and correctly identify multi-axle moving loads even if the middle axle is hanging in the air.
- SVD solution is obviously better than PI, especially for FTDM.
- Identification accuracy increases with mode number. More stations providing high quality responses would be adopted. Error responses at some stations would be appropriately eliminated.
- The vehicle fundamental frequency is varied significantly with the suspension systems. It is evidently beneficial to identification accuracy when suspending and increasing the suspension systems to the non-articulated vehicles .

Thank You

for Your Attendance!